Spring 2011

Design of wind turbine tower and foundation systems: optimization approach

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DESIGN OF WIND TURBINE TOWER AND FOUNDATION SYSTEMS:

OPTIMIZATION APPROACH

by

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A thesis submitted in partial fulfillment
of the requirements for the Master of
Science degree in Civil and Environmental Engineering
in the Graduate College of
The University of Iowa

May 2011

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CERTIFICATE OF APPROVAL

MASTER'S THESIS

This is to certify that the Master's thesis of

John Corbett Nicholson

has been approved by the Examining Committee
for the thesis requirement for the Master of Science
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To my teachers and mentors

ACKNOWLEDGMENTS

I am extremely grateful to Professor Jasbir S. Arora, Professor Colby Swan, Professor Asghar Bhatti, Dr. Marcelo Silva, Provost Barry Butler, and Dr. Tim Marler for their direct support of this work. Professor Jasbir S. Arora not only provided me with the theoretical knowledge of optimization, upon which this work is based, but supported me in obtaining the technical wind turbine tower and foundation design knowledge I would need to bring this work to fruition. Specifically, he invited an expert in the field, Dr. Marcelo Silva, to speak at the University of Iowa and he provided financial support for me to attend a two-day intensive training course on wind turbine tower and foundation system design in Austin Texas. Additionally, Professor Arora's gentle pushing continues to challenge me to be a better student and researcher. As members of my thesis committee and experts in the field of structural engineering, Professors Colby Swan and Asghar Bhatti have been crucial in helping me to ensure that the methodologies and assumptions used in this research are valid. Also, I am very thankful for the time they have taken to review my thesis and provide suggestions to improve it. Their efforts add a great deal to this research and challenge me to think about my research more critically. Dr. Marcelo Silva is thanked for his time and effort in traveling to the University of Iowa, introducing me to the topic of optimization of wind turbine support structures, and suggesting the idea of considering the optimal design of an integral wind turbine tower and foundation system. Provost Barry Butler is thanked for his interest in and feedback on this research throughout the project. Particularly, I am thankful for his efforts to help me partner with those in industry and his insights into the direction of the wind industry. I am thankful and indebted to Dr. Tim Marler for his consistent support of my research and his understanding in allowing me to take the time needed to complete this thesis. This work was supported in part by the Department of Civil and Environmental Engineering and the Center for Computer Aided Design at The University of Iowa.

ABSTRACT

A renewed commitment in the United States and abroad to electricity from renewable resources, such as wind, along with the recent deployment of very large turbines that rise to new heights, makes obtaining the most efficient and safe designs of the structures that support them ever more important. Towards this goal, the present research seeks to understand how optimization concepts and Microsoft Excel's optimization capabilities can be used in the design of wind turbine towers and foundations. Additionally, this research expands on the work of previous researchers to study how considering the tower and foundation as an integral system, where tower support conditions are not perfectly rigid, affects the optimal design. Specifically, optimization problems are formulated and solved with and without taking into account the effect of deflections, resulting from the foundation's rotational and horizontal stiffness, on natural frequency calculations. The general methodology used to transcribe the design of wind turbine towers and foundations into an optimization problem includes: 1) collecting information on design requirements and parameter values 2) deciding how to analyze the structure 3) formulating the optimization problem 4) implementation using Microsoft Excel. Key assumptions include: 1) use of an equivalent lumped mass method for estimating natural frequency 2) International Electrotechnical Commission (IEC) 61400-1 extreme loading condition controls design (i.e. fatigue loading condition is not considered) 3) extreme loads are obtained from manufacturer provided structural load document that satisfies loading cases outlined in IEC 61400-1 4) wind forces on the tower are calculated in accordance with IEC 61400-1 5) optimization variables are continuous. The sum of the tower material and fabrication cost and the total foundation cost is taken as the objective function. Important conclusions from this work include: 1) optimization concepts and Microsoft Excel's optimization capabilities can be used to obtain reasonable conceptual level designs and cost estimates 2) detailed designs and cost

estimates could be achieved using a solver capable of handling discrete optimization problems 3) considering the tower and foundation as an integral system results in a more expensive, but safer, design 4) for the assumed parameter values, the constraint on the tower's natural frequency was found to control the tower design and the bearing capacity constraint was found to control the foundation design 5) relaxing or tightening the limit on the natural frequency will result in the greatest benefit or penalty, respectively, on the optimum solution.

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INTRODUCTION

1.1 Introductory Remarks

Building a clean energy future has been identified as one of the great challenges of our time [Obama 2009]. In order to address this challenge a comprehensive new energy plan was developed in 2009. Part of this plan calls for 10 percent of our electricity to come from renewable resources, such as solar, geothermal and wind, by 2012, and 25 percent by 2025 [Obama 2009]. In addition to this National commitment to electricity from renewable resources such as wind, a recent Harvard University study, published in the Proceedings of the National Academy of Sciences of the United States 2009, estimated world wind power potential to be 40 times greater than total current power consumption [Lu 2009]. This large increase over previous studies, which found this multiple to be closer to 7 times, is in large part due to the increasingly common deployment of very large turbines that rise to heights not considered by previous studies. As turbines rise to new heights, in order to tap into the greater wind speeds available at these heights, obtaining the most efficient and safe or optimal design of the structures that support them will become of increasing importance to the successful proliferation of wind power in the United States and abroad.

Over the past 20 years, a significant amount of research has been conducted to formulate the design of various pole and tower structures as optimization problems. This research has provided us with many valuable insights into the optimal design of such structures as well as into the effectiveness of various optimization approaches. While recent work has been commendable, a gap in the literature surrounding the optimal design of a combined tower and foundation system still exists. Considering the tower and foundation as an integral system, where tower support conditions are not assumed to be perfectly rigid, will allow us to better understand the validity of our current assumptions regarding tower support conditions and to obtain a more accurate wind turbine tower optimal design. Additionally, due to the lack of an official wind turbine tower design standard, previous efforts have relied on a mixture of codes, standards, and engineering judgment to identify design requirements. The present research fills these gaps by incorporating the foundation into the optimum design problem formulation and by adhering to the design guidelines outlined in the recently developed International Electrotechnical Commission (IEC) Wind Turbine Design Requirements document 61400-1.

A review of the literature surrounding the optimal design of various pole and tower structures is now presented in order to show the current state of the art in the field, review various optimization approaches, and identify important assumptions relevant to the present work.

1.2 Review of Literature

Negm and Maalawi 1999 developed and tested six optimization strategies in an effort to obtain the optimal design of a wind turbine tower made up of multiple uniform segments. In all six strategies, each segment's mean diameter, height, and wall-thickness were chosen as design variables. Each strategy differed, however, in selecting a criterion to be optimized: 1) minimization of the tower's mass 2) maximization of the tower's stiffness 3) maximization of the tower's stiffness to mass ratio 4) minimization of vibrations 5) minimization of a performance index that measures the separation between the structure's natural frequency and the turbine's exciting frequency 6) maximization of the system natural frequency. In all six strategies, allowable stress, maximum deflection, resonance, limits on tower mass, limits on mean diameter, and limits on wall-thickness constraints were imposed. The nonlinear programming problem generated in each instance was solved using an interior penalty function optimization method. Important assumptions made in this work include: a) the tower is cantilevered to the ground b) the tower is made up of segments that have different but uniform cross-sectional properties c) the nacelle/rotor unit is treated as a concentrated mass rigidly attached to the tower top d) the tower material is linearly elastic, isotropic, and homogeneous e) the tower cross-section is thin-walled and circular f) deflections are predicted using the Euler-Bernoulli beam theory and secondary effects of axial and shear deformations and rotary inertia are neglected g) distributed aerodynamic loads due to drag forces acting on the tower are considered using a two-dimensional steady flow model h) nonstructural masses, which are distributed along the tower height, are taken as a fraction of the structural mass distribution i) the structural analysis is reduced to a two-dimensional problem such that only bending perpendicular to the plane of the rotor is considered. Of the optimal designs obtained from the six strategies described above, the design obtained by maximizing the system natural frequency yielded the most balanced improvement in both mass and stiffness.

Kocer and Arora 1996 formulated the design of dodecagonal steel transmission poles as an optimization problem. Additionally, they used this formulation to solve an example problem from the literature that had been solved using the conventional design process. The outside diameter of the pole at the tip and the taper of the pole were chosen as continuous design variables. The thickness of the first pole section and the thickness of the second pole section were chosen as discrete design variables. The criterion to be optimized was chosen as the cost of the pole material. Constraints on compressive stress, shear stress, bending stress, combined stress, and deflection were imposed. This discrete optimization problem was solved using multiple optimization methods in the software package IDESIGN 4.2. The two optimization methods that yielded the least expensive results included: 1) a sequential quadratic programming continuous method coupled with a branch and bound discrete method 2) a genetic algorithm that treats all design variables as discrete variables. Important assumptions made in this work include: a) the most conservative design obtained from considering three separate loading cases is taken as the final design b) loads from the conductors, the self-weight of the pole, and the wind action on the pole are considered c) the pole is cantilevered to the ground d) an iterative method is used to calculate secondary moments that result as the center of gravity of the pole shifts due to deflections e) a combined bending stress formula, which incorporates stresses due to moments about the y and z axes, is used to obtain a single design bending stress value f) the design shear stress is taken as the sum of the stresses due to shear and torsion forces g) the design compressive stress is taken as the addition of the axial and bending stresses h) deflections in the y and z directions are calculated by numerically integrating the elastic beam equation i) the design deflection is taken as the square root of the sum of the squares of the deflections in the y and z directions j) the combined stress constraint is formulated using a distortion energy yield criterion. In general, this work showed that the optimal design process can lead to more efficient and safe designs compared to the conventional design process. Additionally, it was demonstrated how, once formulated, an optimization problem can be modified relatively quickly to try different design options and identify the best design (e.g. it was shown that using a circular cross-section resulted in a 2.4% cost savings in material).

Kocer and Arora 1997 extend the work of Kocer and Arora 1996 in an effort to standardize steel pole design by using discrete optimization. In this work, the optimal design is selected from a set of prefabricated pole sections available in a catalog. Additionally, cross-sectional shape and steel grade are added to the list of design variables and welding costs are incorporated into the objective function. Similar constraints to those described in Kocer and Arora 1996 are imposed and a genetic algorithm, an enumeration method, and the simulated annealing method are used to solve the optimization problem. Results from this work showed the enumeration and simulated annealing methods to be significantly more computationally expensive than the genetic algorithm. Additionally, for the case of steel pole design, this study showed that the optimal designs obtained using continuous and discrete methods differed very little. In general, increasing the number of design variables expanded the feasible region which allowed for improvement in the final objective function value.

While studying the optimal design of prestressed concrete transmission poles, Kocer and Arora 1996 showed that the effects of secondary moments, which are due to horizontal deflections, are significant and should be included in pole design calculations. While studying the optimal design of H-frame transmission poles subject to earthquake loadings, Kocer and Arora 1999 found that optimal designs obtained using geometrically nonlinear analysis vs. linear analysis differed very little because other design constraints kept members in the elastic range at the final design. Kocer and Arora 2002 came to a similar conclusion when studying the optimal design of latticed towers subjected to earthquake loading.

Murtagh, Basu, and Broderick 2005 studied the vibration response of wind turbine towers when soil-structure interaction effects are incorporated into the structural model. They found that incorporating the flexibility of soil into the model introduces a considerable amount of damping into the system. This suggests that natural frequency calculations that do not consider damping due to soil-structure interaction will be unrealistic and may lead to uneconomical designs.

Silva, Arora, and Brasil 2008 presented a non-linear model, which was based on experimental data, for the dynamic analysis of reinforced concrete slender structures. Additionally, they transcribed this model into optimization constraints and applied it to the optimal design of reinforced concrete wind turbine towers.

1.3 Objective of Research

The primary objectives of this research are two-fold: 1) to understand how optimization concepts and Microsoft Excel's optimization capabilities can be used in the design of wind turbine towers and foundations. This work will help others to understand how optimization can be applied to the design of wind turbine support structures and to tower structures in general. Furthermore, wide-spread adoption of optimization methods in the design of towers and other structures would lead to more efficient and safe designs 2) to study how considering the tower and foundation as an integral system, where tower support conditions are not perfectly rigid, affects the optimal design. Understanding how the foundation influences the optimal design will help others to make improved assumptions about tower support conditions. This in turn will lead to more efficient, safe, and site specific designs.

1.4 Scope of Thesis

This thesis includes the strategies and methodologies used in the analysis and design, optimization problem formulation, and implementation portions of this research. In addition, it includes worked example problems, a discussion of numerical results, and conclusions. Microsoft Excel's Solver add-in was used to solve example problems.

Chapter II contains the design requirements. It summarizes various criteria that must be considered in the design of wind turbine tower and foundation structures: 1) limits on the tower's cross-sectional dimensions 2) local buckling of the tower wall 3) deflection and rotation of the tower tip 4) limits on the foundation's cross-sectional dimensions 5) bearing capacity of the soil 6) rotational and horizontal stiffness of the foundation 7) overturning of the foundation 8) natural frequency of the tower.

Chapter III contains the analysis. It summarizes the following processes: 1) using the structural load document and information on the wind, self-weight, and internal fixture loads to obtain the tower and foundation loading 2) calculating internal forces, deflections, and stresses in the tower 3) calculating the foundation's bearing on the soil, rotational and horizontal stiffness, and overturning moment 4) obtaining the tower natural frequencies with and without taking into account the effect of the foundational stiffness on the tower's natural frequency.

Chapter IV contains the optimization problem formulation. This chapter defines the various optimization variables, identifies an objective function to be minimized, and transcribes results from the design requirements and analysis chapters into optimization constraints. Types of optimization variables defined include: 1) design variables 2) independent parameters 3) dependent variables 4) height-dependent variables. Types of constraints that are formulated include: 1) design variable constraints 2) natural frequency constraints 3) local buckling constraints 4) tip deflection constraint 5) tip rotation constraint 6) bearing capacity constraints 7) stiffness constraints 8) overturning constraint.

Chapter V summarizes implementation using Excel's Solver add-in and results. Additionally, a brief overview of the Generalized Reduced Gradient optimization method implemented by Excel Solver is given.

Chapter VI contains a discussion of the optimum solutions and the numerical results obtained during solution. This chapter also presents conclusions from the study and areas for further research.

CHAPTER II

DESIGN REQUIREMENTS

2.1 Tower Design Requirements

Cross-sectional dimension, local buckling, and tower top deflection and rotation limits are discussed in this section. Limits on the tower's and combined tower and foundation system's natural frequency are discussed in section 2.3. All of these design requirements must be satisfied in order to ensure a satisfactory tower design.

2.1.1 Cross-Sectional Dimensions

Due to transportation limitations, the outer diameter of the tower cannot exceed 4.5 m. Additionally, due to limitations on the thickness of steel that can be rolled using standard equipment, the maximum tower wall thickness is 40 mm.

2.1.2 Local Buckling

An allowable local buckling stress method and the maximum distortion energy theory are implemented to protect against local buckling of the tower wall.

2.1.2.1 Allowable Local Buckling Stress Method

The allowable local buckling stress method involves [Burton, Sharpe, Jenkins, and Bossanyi 2001]:

1) calculating the elastic critical buckling stress of a cylindrical steel tube, which has modulus of elasticity E_s, wall thickness tw, and mean radius r_m, in axial compression (eqn. 2.2)
2) calculating critical stress reduction coefficients for bending (eqn. 2.3) and axial loading (eqn. 2.4)
3) plugging these values into equation 2.1 along with the material's yield strength f_y to

obtain the allowable local buckling stress. The maximum principal stress in the structure should not exceed this allowable local buckling stress value in order to avoid local buckling.

$$\sigma_{buckling} = \begin{cases} f_y\left[1 - 0.4123\left(\frac{f_y}{\alpha_B\sigma_{cr}}\right)^{0.6}\right] & , \alpha_B\sigma_{cr} > f_y/2 \\ 0.75\alpha_B\sigma_{cr} & , \alpha_B\sigma_{cr} \leq f_y/2 \end{cases} \quad (2.1)$$

$$\sigma_{critical\ elastic} = 0.605\ E_s\frac{tw}{r_m} \quad (2.2)$$

$$\alpha_B = 0.1887 + 0.8113\alpha_0 \quad (2.3)$$

$$\alpha_0 = \begin{cases} \frac{0.83}{\sqrt{1+0.01\ rm/tw}} & , \frac{rm}{tw} < 212 \\ \frac{0.70}{\sqrt{0.1+0.01\ rm/tw}} & , \frac{rm}{tw} \geq 212 \end{cases} \quad (2.4)$$

2.1.2.2 Maximum Distortion Energy Theory

The maximum distortion energy theory states that yielding will occur when the distortion energy per unit volume is equal to that associated with yielding in a simple tension test (eqn. 2.5) [Ugural and Fenster 2003]. This theory is commonly used in engineering design because of its proven track record for predicting failure in ductile materials [Ugural 2003]. Principal stresses σ_1, σ_2, and σ_3 are obtained at the critical points in the tower. In practice, an appropriate factor of safety, FS, is applied to reduce the material's yield stress σ_{yp}.

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\left(\frac{\sigma_{yp}}{FS}\right)^2 \quad (2.5)$$

2.1.3 Tower Top Deflection and Rotation

In order to avoid excessive motion, the maximum deflection at the tower top is limited to 1 and ¼ percent of the tower height. In order to avoid interference between the turbine blades and the tower, the maximum rotation at the tower top is limited to 5°.

2.2 Foundation Design Requirements

Bearing capacity, stiffness, and overturning moment limits are discussed in this section. Settlement is not considered since contact pressures on the soil from vertical loads are typically quite low (e.g. 50 to 75 kPa) in wind turbine foundations and typically cause less than 2.5 cm of settlement in soils having adequate bearing capacity and stiffness [Tinjum and Christensen 2010]. Additionally, detailed structural design, sliding, and liquefaction potential are beyond the scope of the present work.

2.2.1 Bearing Capacity

In order to protect against shear failure in the soil that supports the foundation, the ultimate load that the foundation can sustain, Q_{ult}, must be greater than the total vertical load on the foundation, Q, by an appropriate factor of safety *FS* (eqn. 2.6) [Das 2007]. The process of obtaining this factor of safety for eccentrically loaded foundations, such as those that support wind turbine towers, is known as the Meyerhof or effective area method and involves the following general steps:

1) determine the eccentricity of the loading on the foundation (eqn. 2.7) and use it to calculate the effective width (eqn. 2.8), length (eqn. 2.9), and area (eqn. 2.10) of the foundation
2) calculate Meyerhof's general ultimate bearing capacity (eqn. 2.11), which incorporates
3) calculate the total ultimate load (eqn. 2.23) and ensure that the factor of safety against bearing capacity failure (eqn. 2.6) is sufficient
4) check the factor of safety against the maximum pressure on the soil q_{max} (eqn. 2.24) [Das 2007]. Factors of safety against bearing capacity failure and maximum pressure on the soil typically range from 2 to 3 [Brown 2001].

In the following equations c', q, γ, φ', D_f, and β represent soil cohesion, effective stress at the bottom of the foundation, soil unit weight, soil friction angle, foundation depth, and inclination of the load on the foundation respectively. Also, for circular foundations, B and L are taken as the diameter of the foundation.

$$\mathrm{FS} = \frac{\mathrm{Q_{ult}}}{\mathrm{Q}} \tag{2.6}$$

$$\mathrm{e} = \frac{\mathrm{M}}{\mathrm{Q}} \tag{2.7}$$

$$\mathrm{B'} = \mathrm{B} - 2\mathrm{e} \tag{2.8}$$

$$\mathrm{L'} = \mathrm{L} \tag{2.9}$$

$$\mathrm{A'} = (\mathrm{B'})(\mathrm{L'}) \tag{2.10}$$

$$\mathrm{q'_u} = \mathrm{c'N_cF_{cs}F_{cd}F_{ci}} + \mathrm{qN_qF_{qs}F_{qd}F_{qi}} + \frac{1}{2}\gamma \mathrm{B'N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}} \tag{2.11}$$

$$\mathrm{N_q} = \tan^2\left(45 + \frac{\varphi'}{2}\right) e^{\pi \tan\varphi'} \tag{2.12}$$

$$\mathrm{N_c} = \left(\mathrm{N_q} - 1\right)\cot\varphi' \tag{2.13}$$

$$\mathrm{N_\gamma} = 2\left(\mathrm{N_q} + 1\right)\tan\varphi' \tag{2.14}$$

$$\mathrm{F_{cs}} = 1 + \left(\frac{\mathrm{B'}}{\mathrm{L'}}\right)\left(\frac{\mathrm{N_q}}{\mathrm{N_c}}\right) \tag{2.15}$$

$$\mathrm{F_{qs}} = 1 + \left(\frac{\mathrm{B'}}{\mathrm{L'}}\right)\tan\varphi' \tag{2.16}$$

$$\mathrm{F_{\gamma s}} = 1 - 0.4\left(\frac{\mathrm{B'}}{\mathrm{L'}}\right) \tag{2.17}$$

$$\mathrm{F_{cd}} = \begin{cases} 1 + 0.4\left(\frac{D_f}{B}\right) & , \frac{D_f}{B} \le 1 \\ 1 + (0.4)\tan^{-1}\left(\frac{D_f}{B}\right) & , \frac{D_f}{B} > 1 \end{cases} \tag{2.18}$$

$$\mathrm{F_{qd}} = \begin{cases} 1 + 2\tan\varphi'\,(1 - \sin\varphi')^2\frac{D_f}{B} & , \frac{D_f}{B} \le 1 \\ 1 + 2\tan\varphi'\,(1 - \sin\varphi')^2\tan^{-1}\left(\frac{D_f}{B}\right) & , \frac{D_f}{B} > 1 \end{cases} \tag{2.19}$$

$$\mathrm{F_{\gamma d}} = 1 \tag{2.20}$$

$$\mathrm{F_{ci}} = \mathrm{F_{qi}} = \left(1 - \frac{\beta^\circ}{90^\circ}\right)^2 \tag{2.21}$$

$$\mathrm{F_{\gamma i}} = \left(1 - \frac{\beta}{\varphi'}\right)^2 \tag{2.22}$$

$$Q_{ult} = q'_u A' \tag{2.23}$$

$$FS = \frac{q'_u}{q_{max}} \tag{2.24}$$

2.2.2 Stiffness

In order to avoid excessive motion at the tower top and to provide the required damping, the final foundation design must satisfy minimum rotational and horizontal stiffness values provided by the turbine manufacturer. Typical minimum values of the rotational and horizontal stiffness are 50 GN-m/rad and 1000 MN/m respectively.

2.2.3 Overturning Moment

Since the foundation is subjected to very large moments, the factor of safety against overturning (eqn. 2.25) must be checked [Das 2007]. This factor of safety is obtained by dividing the sum of the moments that tend to resist overturning, $\sum M_R$, by the sum of the moments tending to overturn, $\sum M_O$, the foundation. Moments must be summed about the point where rotation would occur in the event of overturning. This point is usually the toe of the foundation. Factors of safety against overturning typically range from 2 to 3 [Das 2007].

$$FS = \frac{\sum M_R}{\sum M_O} \tag{2.25}$$

2.3 Limit on Natural Frequency

In order to avoid resonance, the natural frequency of the wind turbine support structure must be sufficiently separated from the operating frequency of the turbine. The frequency, in Hertz (Hz), of a particular turbine is obtained by dividing the turbines angular speed in rotations per minute (rpm) by sixty. For utility scale turbines, operating intervals typically range from 14 to 31.4 rpm for the smaller turbines and from 6.2 to 17.7 rpm for the larger turbines. These correspond to operating frequencies between 0.23 and 0.52 Hz for smaller turbines and 0.10 and 0.30 Hz for larger turbines. The natural frequency of the wind turbine support structure must remain above the largest operating frequency of a particular turbine by an appropriate factor, typically between 1.1 and 2, in order to avoid resonance at any point throughout the turbine's operational interval.

CHAPTER III

ANALYSIS

3.1 Tower and Foundation Loads

This section describes the assumed loading on the tower and foundation. The tower loading consists of loads from the turbine, wind, self-weight, and internal fixtures. Loads from the turbine, which act at the tower top, are obtained from a structural load document provided by the turbine manufacturer. Wind, self-weight, and internal fixture loads are obtained using appropriate formulas. Loads on the foundation, which result from the various loads on the tower, are obtained from the structural load document. Additional loading on the foundation includes its self-weight.

3.1.1 Loads from Structural Load Document

This work assumes that tower top and foundation loads are obtained from the structural load document. In order to perform a 2-dimensional analysis, the 3-dimensional sets of force components provided in this document (Figure 3-1) are resolved into 2-dimensional sets of resultant forces that act along newly defined right-handed coordinate systems (Figure 3-2).

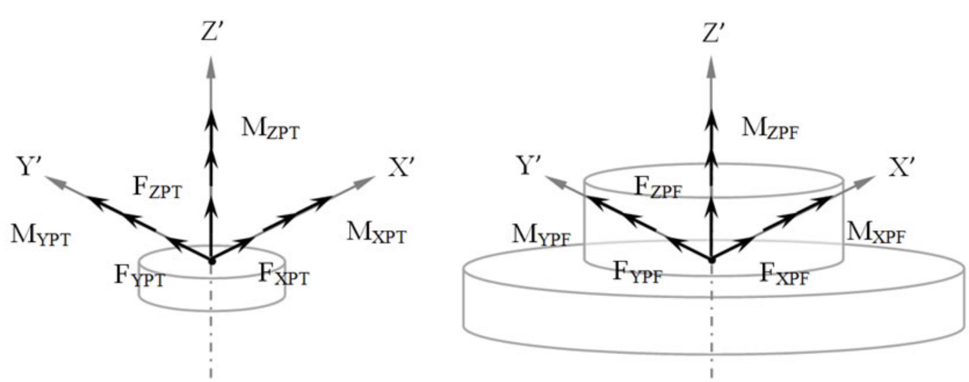

Figure 3-1. 3-dimensional force components at tower top and foundation

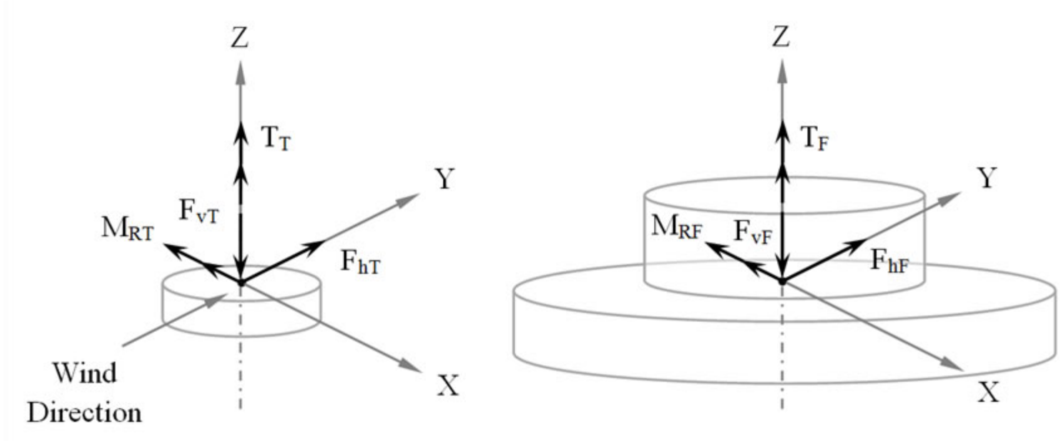

Figure 3-2. 2-dimensional resultant forces at tower top and foundation

Resultant forces acting on tower top:

$$F_{hT} = \sqrt{{F_{XPT}}^2 + {F_{YPT}}^2} \tag{3.1}$$

$$F_{vT} = -F_{ZPT} \tag{3.2}$$

$$M_{RT} = \sqrt{{M_{XPT}}^2 + {M_{YPT}}^2} \tag{3.3}$$

$$T_T = M_{ZPT} \tag{3.4}$$

Resultant forces acting on foundation:

$$F_{hF} = \sqrt{{F_{XPF}}^2 + {F_{YPF}}^2} \tag{3.5}$$

$$F_{vF} = -F_{ZPF} \tag{3.6}$$

$$M_{RF} = \sqrt{{M_{XPF}}^2 + {M_{YPF}}^2} \tag{3.7}$$

$$T_F = M_{ZPF} \tag{3.8}$$

3.1.2 Wind, Self-Weight, and Internal Fixture Loads

In addition to tower top loads, forces due to wind, self-weight, and internal fixtures must be considered in the tower analysis.

Horizontal distributed load due to wind:

$$w_h = c_f \, q_i \, DAF \, d_e(z) \tag{3.9}$$

where c_f is the drag force coefficient, q_i is the wind pressure at height z, DAF is the Dynamic Amplification Factor, and $d_e(z)$ is the exterior diameter at height z.

Vertical distributed load due to tower's self weight and internal fixtures:

$$w_v = w_{IF} + A(z)\gamma \quad (3.10)$$

where w_{IF} is the distributed load due to internal fixtures, $A(z)$ is the cross-sectional area at height z, and γ is the specific weight of the tower material.

The assumed loading for the tower is shown in Figure 3-3.

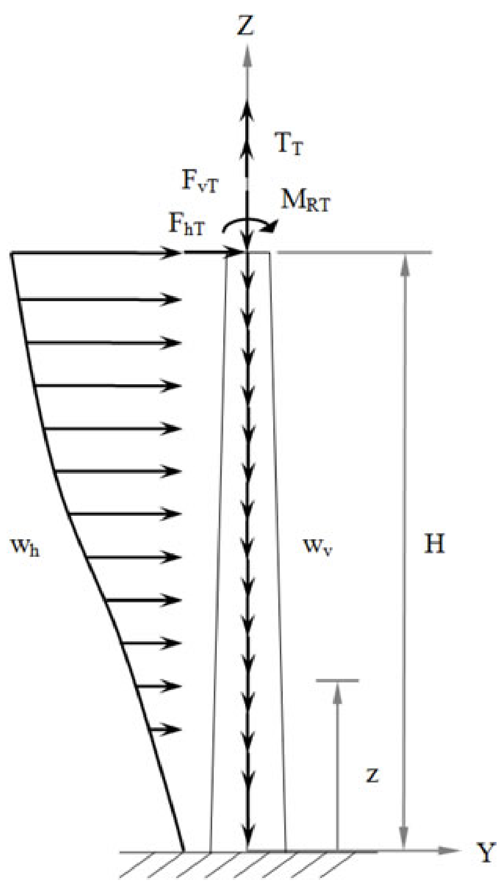

Figure 3-3. Assumed tower loading

In addition to the loads from the structural load document, forces due to the self-weight of the foundation must be considered in the foundation analysis. The self-weight of the foundation acts at its center of gravity in the negative Z-direction.

Vertical load due to self-weight of the foundation:

$$W_f = m_f \, g \tag{3.11}$$

where m_f is the mass of the foundation and g is the gravitational constant.

3.2 Tower Analysis

The tower analysis consists of calculating the internal forces, deflections, second order moments due to deflections, stress components, and principal stresses. Stress components and corresponding principal stresses are obtained at critical points and the maximum principal stress is taken as the design stress.

3.2.1 Internal Forces

In order to determine the internal forces acting in the tower, a section is cut at a distance y from the tower top, an appropriate free body diagram is drawn (Figure 3-4), and the internal forces are calculated.

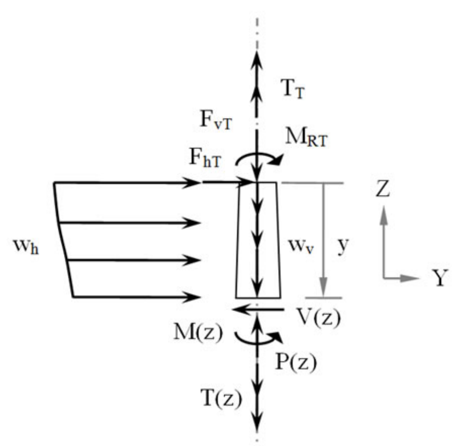

Figure 3-4. Free body diagram of tower section

Internal forces as a function of height z:

$$P(z) = F_{vT} + \int_0^{H-z} w_v dy \tag{3.12}$$

$$V(z) = F_{hT} + \int_0^{H-z} w_h dy \tag{3.13}$$

$$T(z) = T_T \tag{3.14}$$

$$M(z) = M_{RT} + F_{hT}(H - z) + \int_0^{H-z} w_h (H - z - y) dy + M(z)_{2nd\ Order} \tag{3.15}$$

Second order moments, which result from deflections, are determined using an iterative procedure and equation 3.16.

$$M(z)_{2nd\ Order} = P(z) \times 0.75 \times (v_{top} - v_i) \tag{3.16}$$

where v_i is the deflection of the i^{th} section and v_{top} is the deflection of the tower top.

3.2.2 Deflections

Deflections are determined by implementing a numerical integration procedure, which uses the trapezoidal rule, on the differential elastic line equation twice. It is important to note that this equation assumes linearly elastic behavior of the structure.

$$EI(z)v''(z) = -M(z) \tag{3.17}$$

$$v_i'' = -\frac{M(z_i)}{EI(z_i)} \tag{3.18}$$

$$v_i' = v_{i-1}' + \frac{v_i'' + v_{i-1}''}{2} h \tag{3.19}$$

$$v_i = v_{i-1} + \frac{v_i' + v_{i-1}'}{2} h \tag{3.20}$$

where E is the young's modulus, $I(z)$ is the cross-section moment of inertia at height z, and $h = z_i - z_{i-1}$.

3.2.3 Stresses at Cross-Section

Once the internal forces acting in the tower are determined, the internal stress components and principal stresses at the two critical points shown in Figure 3-5 are calculated. The maximum principal stress is taken as the design stress.

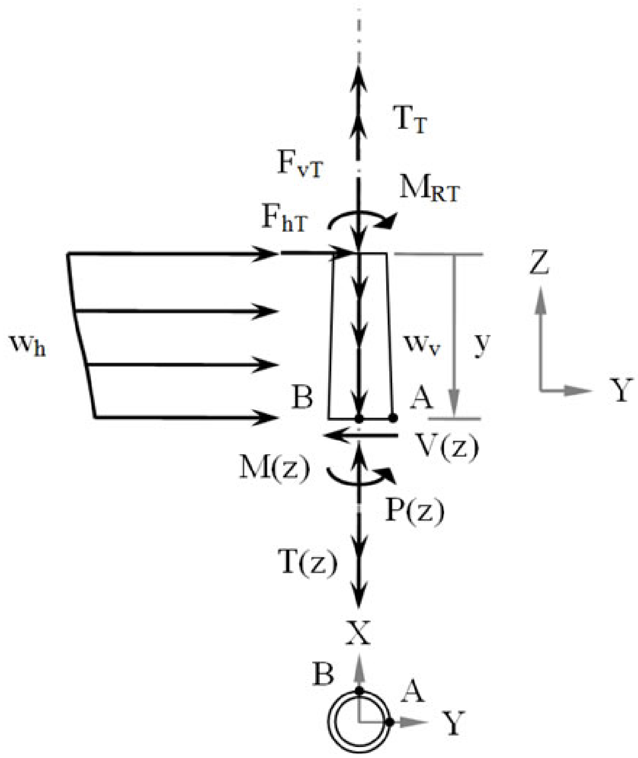

Figure 3-5. Location of critical points A and B

3.2.3.1 Stress Components

Internal stress components at critical point A:

$$\sigma_Z = \frac{P(z)}{A(z)} + \frac{M(z)\left(\frac{d_e(z)}{2}\right)}{I(z)} \tag{3.21}$$

$$\tau_{ZX} = \frac{T(z)\left(\frac{d_e(z)}{2}\right)}{J(z)} \tag{3.22}$$

Internal stress components at critical point B:

$$\sigma_Z = \frac{P(z)}{A(z)} \tag{3.23}$$

$$\tau_{ZY} = \frac{T(z)\left(\frac{d_e(z)}{2}\right)}{J(z)} + \frac{V(z)\,Q(z)}{I(z)\,b(z)} \tag{3.24}$$

where $A(z)$ is the cross-sectional area at height z, $d_e(z)$ is the exterior diameter of the tower at height z, $J(z)$ is the polar moment of inertia at height z, $Q(z)$ is the first moment at height z, and $b(z)$ is the cross-sectional material width at height z.

3.2.3.2 Principal Stresses

Principal stresses (i.e. σ_1, σ_2, and σ_3) at critical points A and B are the eigenvalues (i.e. σ_p) of the stress tensor, which can be found by solving the characteristic equation (i.e. eqn. 3.25), at each critical point [Ugural and Fenster 2003]:

$$\begin{vmatrix} \sigma_X - \sigma_p & \tau_{XY} & \tau_{XZ} \\ \tau_{XY} & \sigma_Y - \sigma_p & \tau_{YZ} \\ \tau_{XZ} & \tau_{YZ} & \sigma_Z - \sigma_p \end{vmatrix} = 0 \qquad (3.25)$$

where $\sigma_1 \geq \sigma_2 \geq \sigma_3$.

3.3 Foundation Analysis

The foundation analysis consists of calculating the total vertical load, maximum load on the soil, foundation stiffness values, and foundation overturning and resisting moments. These values will be required in order to ensure that the foundation design requirements in chapter 2 are satisfied.

3.3.1 Total Vertical Load

The total vertical load is the sum of the vertical force on the foundation from the structural load document and the self-weight of the foundation (eqn. 3.26).

$$Q = F_{vF} + W_f \qquad (3.26)$$

3.3.2 Maximum Pressure on Soil

The maximum pressure on the soil for eccentrically loaded foundations is given by equation 3.27 [Das 2007]. The second case, $e \geq B/6$, gives the maximum pressure on the soil when a separation between the foundation and the soil underlying it occurs. Eccentricity of the loading is calculated using equation 3.28 [Das 2007].

$$q_{max} = \begin{cases} \frac{Q}{BL}\left(1 + \frac{6e}{B}\right) & , \ e < \frac{B}{6} \\ \frac{4Q}{3L(B-2e)} & , \ e \geq \frac{B}{6} \end{cases} \tag{3.27}$$

$$e = \frac{M_{RF}}{Q} \tag{3.28}$$

where B and L are taken as the diameter for a circular foundation.

3.3.3 Foundation Stiffness

Foundation stiffness values depend on properties of the soil, dimensions of the foundation, and the depth to bedrock. The rotational stiffness is the ratio of the overturning moment to the rotation angle. The horizontal stiffness is the ratio between the horizontal force and the horizontal displacement. Rotational and horizontal stiffness values for a circular footing embedded in a stratum over bedrock are given in equations 3.29 and 3.30 respectively [Riso and DNV 2002].

$$K_R = \frac{8GR^3}{3(1-v)}\left(1 + \frac{R}{6H_b}\right)\left(1 + 2\frac{D_f}{R}\right)\left(1 + 0.7\frac{D_f}{H_b}\right) \tag{3.29}$$

$$K_H = \frac{8GR}{1-v}\left(1 + \frac{R}{2H_b}\right)\left(1 + \frac{2}{3}\frac{D_f}{R}\right)\left(1 + \frac{5}{4}\frac{D_f}{H_b}\right) \tag{3.30}$$

where G, R, v, D_f, and H_b are the shear modulus of the soil, radius of the foundation, Poisson's ratio of the soil, depth of the foundation, and depth to bedrock respectively.

3.3.4 Foundation Overturning

Resisting (eqn. 3.31) and overturning (eqn. 3.32) moments used in determining the factor of safety against overturning are calculated about the toe of the foundation since this is where rotation would take place in the event of overturning [Das 2007]. The total resisting moment is the total vertical load multiplied by one-half of the foundation's diameter. The total overturning moment is the sum of the resultant moment and the resultant horizontal force multiplied by the height of the foundation.

$$\sum M_R = Q \times \frac{B}{2} \tag{3.31}$$

$$\sum M_O = F_{hF} \times \left(D_f + h_p\right) + M_{RF} \tag{3.32}$$

where D_f and h_p are the foundation depth and height of the pedestal above grade respectively.

3.4 Frequency Analysis

The natural frequency, f_n, of the tower and the combined tower and foundation system is obtained using the equivalent lumped mass method that derives from Rayleigh's method. This method assumes that the mass of the tower is lumped at a series of points and that the deflection under static loading is close to the fundamental mode shape. The method is implemented by calculating effective stiffness (eqn. 3.27) and effective mass (eqn. 3.28) values and plugging them into equation 3.26.

$$f_n = \frac{1}{2\pi}\sqrt{\frac{\bar{k}}{\bar{m}}} \tag{3.33}$$

$$\bar{k} = g\sum_{i=1}^{n} m_i v_i \tag{3.34}$$

$$\bar{m} = g\sum_{i=1}^{n} m_i v_i^2 \tag{3.35}$$

where m_i is the mass of tower section *i* and v_i is the deflection of tower section *i* under static loading.

CHAPTER IV

OPTIMIZATION PROBLEM FORMULATION

4.1 Optimization Variables

Variables used in the optimization problem formulation include design variables, independent parameters, dependent variables, and height dependent variables.

4.1.1 Design Variables

Cross-sectional dimensions of the tower and foundation are treated as design variables in the optimization problem formulation:

d_{bo} = outer diameter of the tower base

d_{to} = outer diameter of the tower top

t = thickness of the tower wall

B = diameter of the foundation base

t_{fo} = thickness of foundation at its outer edge

4.1.2 Independent Parameters

The following parameters are independent of the design variables and require specification before the optimization process can begin:

F_{XPT} = horizontal force at the tower top in line with the rotor axis

F_{YPT} = horizontal sideways force at the tower top

F_{ZPT} = vertical upwards force at the tower top

M_{XPT} = moment along X′ axis at the tower top

M_{YPT} = moment along Y′ axis at the tower top

M_{ZPT} = moment along Z′ axis at the tower top

F_{XPF} = horizontal force at the foundation in line with the rotor axis

F_{YPF} = horizontal sideways force at the foundation

F_{ZPF} = vertical upwards force at the foundation

M_{XPF} = moment along X′ axis at the foundation

M_{YPF} = moment along Y′ axis at the foundation

M_{ZPF} = moment along Z′ axis at the foundation

F_{hT} = horizontal resultant force at the tower top

F_{vT} = vertical downward force at the tower top

M_{RT} = resultant moment at the tower top

T_T = torque at the tower top

F_{hF} = horizontal resultant force at the foundation

F_{vF} = vertical downward force at the foundation

M_{RF} = resultant moment at the foundation

T_F = torque at the foundation

ρ_s = mass density of steel

μ_t = tower specific cost

μ_f = foundation specific cost

$FS_{frequency}$ = factor of safety for natural frequency

$freq$ = turbine frequency

f_y = yield strength for steel

FS_{MDE} = maximum distortion energy factor of safety

v_a = allowable tip deflection

v_a' = allowable tip rotation

K_{ReqRot} = minimum required rotational stiffness

$K_{ReqHoriz}$ = minimum required horizontal stiffness

V_{ref} = reference wind speed

H_n = nacelle height

W_n = nacelle width

H = height of the tower

ρ_a = density of air

γ_s = specific weight of steel

ρ_c = mass density of concrete

E_s = modulus of elasticity for steel

c_f = drag force coefficient for tower

DAF = Dynamic Amplification Factor

w_{IF} = vertical distributed load due to internal fixtures

g = gravitational constant

φ' = soil friction angle

c' = soil cohesion

γ = soil unit weight

t_p = pedestal thickness

d_p = pedestal diameter

a_f = foundation dimension a

h_p = pedestal height above grade

G_o = initial shear modulus

γ_{shear} = shear modulus reduction ratio

G = shear modulus

v = Poisson's ratio

H_{hub} = hub height

τ_{yield} = available shear yield strength

N_q = bearing capacity factor

N_c = bearing capacity factor

N_γ = bearing capacity factor

β = inclination of the load on the foundation with respect to vertical

F_{ci} = inclination factor for ultimate bearing capacity calculation

F_{qi} = inclination factor for ultimate bearing capacity calculation

$F_{\gamma i}$ = inclination factor for ultimate bearing capacity calculation

4.1.3 Dependent Variables

The following variables are dependent on the design variables and will change throughout the optimization process:

τ = taper of the tower

V_t = volume of the tower

m_t = mass of the tower

V_f = volume of the foundation

m_f = mass of the foundation

f_n = natural frequency

$\sigma_1^{cpA\_t}$ = maximum principal stress at critical point A at tower top

$\sigma_2^{cpA\_t}$ = intermediate principal stress at critical point A at tower top

$\sigma_3^{cpA\_t}$ = minimum principal stress at critical point A at tower top

$\sigma_1^{cpB\_t}$ = maximum principal stress at critical point B at tower top

$\sigma_2^{cpB\_t}$ = intermediate principal stress at critical point B at tower top

$\sigma_3^{cpB\_t}$ = minimum principal stress at critical point B at tower top

$\sigma_1^{cpA\_b}$ = maximum principal stress at critical point A at tower base

$\sigma_2^{cpA\_b}$ = intermediate principal stress at critical point A at tower base

$\sigma_3^{cpA\_b}$ = minimum principal stress at critical point A at tower base

$\sigma_1^{cpB\_b}$ = maximum principal stress at critical point B at tower base

$\sigma_2^{cpB\_b}$ = intermediate principal stress at critical point B at tower base

$\sigma_3^{cpB\_b}$ = minimum principal stress at critical point B at tower base

r_m = mean cross-sectional radius

$\sigma_{CriticalElastic}$ = elastic critical buckling stress

α_0 = axial critical stress reduction coefficient

α_B = bending critical stress reduction coefficient

$\sigma_{buckling}$ = allowable local buckling stress

v_{top} = tip deflection at tower top

v'_{top} = tip rotation at tower top

D_f = foundation depth

q = equivalent surcharge for soil above the foundation

b_f = foundation dimension b

e = loading eccentricity of the foundation

b_e = foundation effective area minor axis

I_e = foundation effective area major axis

A_{eff} = foundation effective area

L' = effective length of the foundation

B' = effective width of the foundation

A' = effective area of equivalent rectangle

F_{cs} = shape factor for ultimate bearing capacity calculation

F_{qs} = shape factor for ultimate bearing capacity calculation

$F_{\gamma s}$ = shape factor for ultimate bearing capacity calculation

F_{cd} = depth factor for ultimate bearing capacity calculation

F_{qd} = depth factor for ultimate bearing capacity calculation

$F_{\gamma d}$ = depth factor for ultimate bearing capacity calculation

q'_u = Meyerhof ultimate bearing capacity

Q_{ult} = total ultimate load the foundation can sustain

Q_{total} = total vertical load on the foundation

$FS_{bearing}$ = factor of safety against bearing capacity failure

q_{max} = maximum pressure on soil

$FS_{SoilPressure}$ = factor of safety for maximum pressure on soil

K_R = rotational stiffness of the foundation

K_H = horizontal stiffness of the foundation

$\sum M_R$ = sum of resisting moments

$\sum M_O$ = sum of overturning moments

4.1.4 Height Dependent Variables

The following variables change with the height of the tower:

z = height above ground

V_{e50} = expected extreme wind speed with recurrence interval of 50 years

q_i = wind pressure

d_o = outer diameter of tower

w_h = horizontal distributed wind load

d_i = inner diameter of tower

A = cross-sectional area of tower

w_v = vertical distributed load

y = distance from tower top

P = axial force at cross section

V = shear force at cross section

T = torque at cross section

M = moment at cross section

I = moment of inertia at cross section

v''= curvature in the vertical plane

v'= rotation in the vertical plane

v = horizontal displacement

M_{2nd} = 2nd order moments

σ_{ZcpA} = compressive stress along the z-axis at critical point A

σ_{ZcpB} = compressive stress along the z-axis at critical point B

J = polar moment of inertia at cross section

τ_{ZXcpA} = shear stress due to torque at critical point A

Q = 1st moment of inertia at cross section

b = width where shear stress is calculated

τ_{ZYcpB} = shear stress due to torque and shear at critical point B

σ_1^{cpA} = maximum principal stress at critical point A

σ_2^{cpA} = intermediate principal stress at critical point A

σ_3^{cpA} = minimum principal stress at critical point A

σ_1^{cpB} = maximum principal stress at critical point B

σ_2^{cpB} = intermediate principal stress at critical point B

σ_3^{cpB} = minimum principal stress at critical point B

$\bar{k}$ = effective stiffness for natural frequency calculation

$\bar{m}$ = effective mass for natural frequency calculation

4.2 Objective Function

Our objective function is to minimize the tower and foundation cost (i.e. eqn. 4.1). Calculating the objective function involves multiplying the masses of the tower and foundation by their respective specific cost values and summing.

$$f = m_t \times \mu_t + m_f \times \mu_f \tag{4.1}$$

4.3 Constraints

In this section, design requirements outlined in chapter 2 are transcribed into optimization constraints. Specifically, design requirements related to limits on the design variables, limits on the natural frequency, local buckling limits, tip deflection and rotation limits, allowable bearing capacity, foundation stiffness, and foundation overturning are converted into inequality constraints, which are represented by the letter g, and written in the standard form. Also, constraints have been normalized, where appropriate, in order to improve the optimization process [Arora 2004].

4.3.1 Design Variable Constraints

Constraints on the tower design variables, which define the final tower design, come from transportation and manufacturing limitations. Constraints on the foundation design variables, which define the final foundation design, come from maximum and minimum values typically obtained when designing shallow wind turbine foundations.

4.3.1.1 Limits on Outer Diameter of Tower Base

As discussed in chapter 2, the outer diameter of the tower base cannot exceed 4.5 m due to transportation limitations. Additionally, we must set a lower bound on the outer diameter of the tower base to ensure that this value remains above zero during the optimization process.

$$g_1 = 0.1 - d_{bo} \leq 0 \qquad (4.2)$$

$$g_2 = d_{bo} - 4.5 \leq 0 \qquad (4.3)$$

4.3.1.2 Limits on Outer Diameter of Tower Top

The outer diameter of the tower top cannot exceed the width of the nacelle. For the turbine being considered this value is 3.4 m. Also, we must once again set a lower bound on the outer diameter to ensure that this value remains above zero during the optimization process.

$$g_3 = 0.1 - d_{to} \leq 0 \qquad (4.4)$$

$$g_4 = d_{to} - 3.4 \leq 0 \qquad (4.5)$$

4.3.1.3 Limits on Tower Wall Thickness

As discussed in chapter 2, due to limitations on the thickness of steel that can be rolled using standard equipment, the maximum tower wall thickness is 40 mm. Similar to above, a lower bound on the wall thickness is imposed to ensure that this value remains above zero during the optimization process.

$$g_5 = 0.001 - t \leq 0 \qquad (4.6)$$

$$g_6 = t - 0.04 \leq 0 \qquad (4.7)$$

4.3.1.4 Limits on Diameter of Foundation

Typical shallow wind turbine foundation designs have base diameters ranging from 10 to 30 m. Therefore, lower and upper limits of 10 and 30 m respectively are placed on the foundation's base diameter.

$$g_7 = 10 - B \leq 0 \quad (4.8)$$

$$g_8 = B - 30 \leq 0 \quad (4.9)$$

4.3.1.5 Limits on Thickness of Foundation at Outer Edge

Typical shallow wind turbine foundation designs have outer edge thicknesses ranging from 0.50 to 1.5 m. Therefore, lower and upper limits of 0.50 and 1.5 m respectively are placed on the foundation's outer edge thickness.

$$g_9 = 0.50 - t_{fo} \leq 0 \quad (4.10)$$

$$g_{10} = t_{fo} - 1.5 \leq 0 \quad (4.11)$$

4.3.2 Natural Frequency Constraint

As discussed in chapter 2, the natural frequency of the wind turbine support structure must remain above the largest operating frequency of a particular turbine by an appropriate factor, typically between 1.1 and 2, in order to avoid resonance at any point throughout the turbine's operational interval. This constraint has been normalized by dividing both sides of the inequality by the tower's natural frequency.

$$g_{11} = \frac{FS_{frequency} \times freq}{f_n} \leq 1 \quad (4.12)$$

4.3.3 Local Buckling Constraints

An allowable local buckling stress method and the maximum distortion energy theory are implemented to protect against local buckling of the tower wall. This section describes the transcription of these methods into optimization constraints.

4.3.3.1 Allowable Local Buckling Stress

The allowable local buckling stress method requires that the maximum principal stress in the structure not exceed an allowable local buckling stress. This limit is imposed at the tower top and base in order to ensure that this condition is satisfied over the entire height of the structure. Additionally, maximum principal stress values are calculated at both of our critical points, A and B, to account for the stress variation over the cross-section.

$$g_{12} = \frac{\sigma_1^{cpA\_t}}{\sigma_{buckling}} \leq 1 \tag{4.13}$$

$$g_{13} = \frac{\sigma_1^{cpA\_b}}{\sigma_{buckling}} \leq 1 \tag{4.14}$$

$$g_{14} = \frac{\sigma_1^{cpB\_t}}{\sigma_{buckling}} \leq 1 \tag{4.15}$$

$$g_{15} = \frac{\sigma_1^{cpB\_b}}{\sigma_{buckling}} \leq 1 \tag{4.16}$$

4.3.3.2 Maximum Distortion Energy

The maximum distortion energy theory requires that the distortion energy per unit volume in the structure not exceed that associated with yielding in a simple tension test. An appropriate factor of safety, FS_{MDE}, is typically applied to reduce the material's yield stress f_y. This limit is imposed at the tower top and base in order to ensure that this condition is satisfied over the entire height of the structure. Additionally, maximum distortion energy values are calculated at both of our critical points, A and B, to account for the stress variation over the cross-section.

$$g_{16} = \frac{\left(\sigma_1^{cpA\_t} - \sigma_2^{cpA\_t}\right)^2 + \left(\sigma_2^{cpA\_t} - \sigma_3^{cpA\_t}\right)^2 + \left(\sigma_3^{cpA\_t} - \sigma_1^{cpA\_t}\right)^2}{2\left(\frac{f_y}{FS_{MDE}}\right)^2} \leq 1 \tag{4.17}$$

$$g_{17} = \frac{\left(\sigma_1^{cpA\_b} - \sigma_2^{cpA\_b}\right)^2 + \left(\sigma_2^{cpA\_b} - \sigma_3^{cpA\_b}\right)^2 + \left(\sigma_3^{cpA\_b} - \sigma_1^{cpA\_b}\right)^2}{2\left(\frac{f_y}{FS_{MDE}}\right)^2} \leq 1 \tag{4.18}$$

$$g_{18} = \frac{\left(\sigma_1^{cpB\_t} - \sigma_2^{cpB\_t}\right)^2 + \left(\sigma_2^{cpB\_t} - \sigma_3^{cpB\_t}\right)^2 + \left(\sigma_3^{cpB\_t} - \sigma_1^{cpB\_t}\right)^2}{2\left(\frac{fy}{FS_{MDE}}\right)^2} \leq 1 \tag{4.19}$$

$$g_{19} = \frac{\left(\sigma_1^{cpB\_b} - \sigma_2^{cpB\_b}\right)^2 + \left(\sigma_2^{cpB\_b} - \sigma_3^{cpB\_b}\right)^2 + \left(\sigma_3^{cpB\_b} - \sigma_1^{cpB\_b}\right)^2}{2\left(\frac{fy}{FS_{MDE}}\right)^2} \leq 1 \tag{4.20}$$

4.3.4 Tip Deflection Constraint

As discussed in chapter 2, the maximum deflection at the tower top is limited to an allowable deflection of 1 and ¼ percent of the tower height in order to avoid excessive motion of the turbine.

$$g_{20} = \frac{v_{top}}{v_a} \leq 1 \tag{4.21}$$

4.3.5 Tip Rotation Constraint

As discussed in chapter 2, the maximum rotation at the tower top is limited to an allowable rotation of 5° in order to avoid interference between the turbine blades and the tower.

$$g_{21} = \frac{v'_{top}}{v'_a} \leq 1 \tag{4.22}$$

4.3.6 Bearing Capacity Constraints

In order to protect against shear failure in the soil that supports the foundation, the ultimate load that the foundation can sustain must be greater than the total vertical load on the foundation by an appropriate factor of safety $FS_{bearing}$. Additionally, the ultimate bearing pressure that the foundation can sustain must be greater than the maximum pressure on the soil by an appropriate factor of safety $FS_{SoilPressure}$. Factors of safety against bearing capacity failure and maximum pressure on the soil typically range from 2 to 3. In the present work, a value of 3 is used in order to ensure a conservative design.

$$g_{22} = \frac{3}{FS_{bearing}} \leq 1 \tag{4.23}$$

$$g_{23} = \frac{3}{FS_{SoilPressure}} \leq 1 \quad (4.24)$$

4.3.7 Stiffness Constraints

A discussed in chapter 2, the final foundation design must satisfy minimum rotational and horizontal stiffness values provided by the turbine manufacturer in order to avoid excessive motion at the tower top and to provide the required damping. Typical minimum values of the rotational and horizontal stiffness are 50 GN-m/rad and 1000 MN/m respectively.

$$g_{24} = \frac{K_{ReqRot}}{K_R} \leq 1 \quad (4.25)$$

$$g_{25} = \frac{K_{ReqHoriz}}{K_H} \leq 1 \quad (4.26)$$

4.3.8 Overturning Constraint

The factor of safety against overturning, which is obtained by dividing the sum of the moments that tend to resist overturning (i.e. $\sum M_R$) by the sum of the moments tending to overturn (i.e. $\sum M_O$) the foundation, typically ranges from 2 to 3. Since the loads that cause the resisting and overturning moments on our structure are well defined, a factor of safety on the lower end of the typical range is used.

$$g_{26} = \frac{2 \times \sum M_O}{\sum M_R} \leq 1 \quad (4.27)$$

CHAPTER V

IMPLEMENTATION AND RESULTS

5.1 Implementation using Excel Solver

Microsoft Excel's Solver add-in was used to implement the optimization problem formulated in chapter 4. However, before implementation could occur, it was necessary to define all design variables, independent parameters, dependent variables, height dependent variables, constraints, and objective function information in an Excel workbook.

The initial values of the design variables, which are shown in table 5.1, were taken as the average of the lower and upper limit on each design variable. The values of the independent parameters used in the solution process have been discussed in previous chapters and are summarized in table 5.2. Dependent variable values have been calculated using formulas presented in chapters 2 and 3 and their values at the optimal solution are shown in table 5.3. Similarly, height dependent variables have been calculated using formulas in chapters 2 and 3 and their final values at the tower top and base are shown in table 5.4.

As shown in chapter 4, constraints on the design variables have not been normalized. This practice was followed when defining constraints on the design variables in Excel. All other constraints have been normalized in both chapter 4 and Excel. Also, all constraints have been formulated in the standard form as less than or equal to type. This facilitates efficient entry into Excel Solver. Last, our objective function is defined and Excel Solver is invoked.

In the Solver Parameters dialog box our optimization problem is defined. The "Target Cell" is set to the cell containing the formula for our objective function and the "Min" radio button is selected to tell Solver to minimize our objective function. The cells containing the design variable values are input into the "By Changing Cells" box.

Similarly, the constraints are defined in the "Subject to the Constraints" box. After defining the problem in the Solver Parameters dialog box, numerical parameters used in the optimization solution process can be defined by clicking "Options".

The resulting Solver Options dialog box allows the user to specify the maximum time to allow the solution process to run, the maximum number of iterations to allow the solution process to complete, the precision of the solution, the tolerance on the constraints, and the acceptable convergence of the algorithm. The user may also specify other parameters such as the use of forward or central difference methods for calculating derivatives and Newton or Conjugate methods for calculating search directions. Specific values and selections used in this work are as follows: 1) the maximum amount of time to run the solution process is set to 100-seconds 2) the maximum number of iterations is set to 10,000 3) the precision is set to 0.000001 4) the tolerance on the constraints is set to 1% 5) convergence is set to 0.00001 5) "Use Automatic Scaling" is selected to improve numerical stability 6) derivatives are calculated using the central difference method 7) Newton's method is used for search direction calculations.

Once the problem has been set up and defined, the Solver can be initiated and the optimal solution obtained. Answer, sensitivity, and limits reports can also be generated at this time. The contents of these reports are summarized in tables 5.5 through 5.8. The significance of the values contained in these tables is discussed in chapter 6.

5.2 Generalized Reduced Gradient (GRG) Method

Excel Solver uses a version of the Generalized Reduced Gradient (GRG) optimization method for solving nonlinear problems. Specifically, the GRG2 code, which was developed by Lasdon and Waren, is used. The GRG method is an extension of the reduced gradient method, which is an algorithm for quadratic programming, to handle nonlinear inequality constraints [Arora 2004]. The general idea of the method is to find a search direction such that the current active constraints remain precisely active for any small move and to use the Newton Raphson method to return to the constraint boundary when active constraints are not precisely satisfied [Arora 2004]. Excel Solver's GRG implementation offers both Conjugate Gradient and Newton based methods for determining a search direction. In the present work, Newton's method has been used.

5.3 Results

5.3.1 Optimal Solution

Table 5.1 shows the optimal designs obtained when considering the tower and foundation as individual components and as an integral system. To analyze the tower and foundation as an integral system the foundational stiffness is taken into account when calculating the tower's natural frequency.

Table 5.1 Optimal Design of Wind Turbine Tower and Foundation

	Tower and Foundation		Tower and Foundation System	
Variable	Initial Guess	Optimal Value	Initial Guess	Optimal Value
d_{bo} [m]	2.3	4.5	2.3	4.5
d_{to} [m]	1.75	3.4	1.75	3.4
t [mm]	20.5	35.3	20.5	35.4
B [m]	20	11.84	20	11.84
t_{fo} [m]	1.0	0.5	1.0	0.5
f [$]	473,155	523,772	473,155	524,918

Table 5.2 shows values of the independent parameters used to describe the problem.

Table 5.2 Independent Parameter Values

Parameter (Symbol) [Units]	Value
Horizontal force at the tower top in line with the rotor axis (F_{XPT}) [kN]	809
Horizontal sideways force at the tower top (F_{YPT}) [kN]	47.3
Vertical upwards force at the tower top (F_{ZPT}) [kN]	-1,342
Moment along X′ axis at the tower top (M_{XPT}) [kN-m]	1,639
Moment along Y′ axis at the tower top (M_{YPT}) [kN-m]	2,179
Moment along Z′ axis at the tower top (M_{ZPT}) [kN-m]	2,499
Horizontal force at the foundation in line with the rotor axis (F_{XPF}) [kN]	1,245
Horizontal sideways force at the foundation (F_{YPF}) [kN]	389
Vertical upwards force at the foundation (F_{ZPF}) [kN]	-5,420
Moment along X′ axis at the foundation (M_{XPF}) [kN-m]	11,542
Moment along Y′ axis at the foundation (M_{YPF}) [kN-m]	16,303
Moment along Z′ axis at the foundation (M_{ZPF}) [kN-m]	2,499
Horizontal resultant force at the tower top (F_{hT}) [kN]	810.4
Vertical downward force at the tower top (F_{vT}) [kN]	1,342
Resultant moment at the tower top (M_{RT}) [kN-m]	2,727
Torque at the tower top (T_T) [kN-m]	2,499
Horizontal resultant force at the foundation (F_{hF}) [kN]	1,304
Vertical downward force at the foundation (F_{vF}) [kN]	5,420
Resultant moment at the foundation (M_{RF}) [kN-m]	19,975
Torque at the foundation (T_F) [kN-m]	2,499
Mass density of steel (ρ_s) [kg/m^3]	7850
Tower specific cost (μ_t) [$/kg]	1.5
Foundation specific cost (μ_f) [$/kg]	0.256
Factor of safety for natural frequency ($FS_{frequency}$)	2
Turbine frequency ($freq$) [Hz]	0.33
Yield strength for steel (f_y) [MPa]	345
Maximum distortion energy factor of safety (FS_{MDE})	1.2
Allowable tip deflection (v_a) [m]	1
Allowable tip rotation (v_a') [deg]	5
Minimum required rotational stiffness (K_{ReqRot}) [GN-m/rad]	50
Minimum required horizontal stiffness ($K_{ReqHoriz}$) [MN/m]	1,000
Reference wind speed (V_{ref}) [m/s]	50

Table 5.2 continued

Nacelle height (H_n) [m]	4
Nacelle width (W_n) [m]	3.4
Height of the tower (H) [m]	80
Density of air (ρ_a) [kg/m^3]	1.225
Specific weight of steel (γ_s) [N/m^3]	77,000
Mass density of concrete (ρ_c) [kg/m^3]	2,400
Modulus of elasticity for steel (E_s) [GPa]	210
Drag force coefficient for tower (c_f)	0.6
Dynamic Amplification Factor (DAF)	1.11451
Vertical distributed load due to internal fixtures (w_{IF}) [kN/m]	0.8
Gravitational constant (g) [m/s^2]	9.81
Soil friction angle (φ') [deg]	10
Soil cohesion (c') [kN/m^2]	15.2
Soil unit weight (γ) [kN/m^3]	17.8
Pedestal thickness (t_p) [m]	1.5
Pedestal diameter (d_p) [m]	5.6
Foundation dimension a (a_f) [m]	2.8
Pedestal height above grade (h_p) [m]	0.1524
Initial shear modulus (G_o) [MN/m^2]	44.65
Shear modulus reduction ratio (γ_{shear})	0.6
Height to bedrock (H_b) [m]	10
Shear modulus (G) [MN/m^2]	180
Poisson's ratio (υ)	0.5
Hub height (H_{hub}) [m]	82
Available shear yield strength (τ_{yield}) [MPa]	207
Bearing capacity factor (N_q)	2.47
Bearing capacity factor (N_c)	8.35
Bearing capacity factor (N_γ)	1.22
Inclination of the load on the foundation with respect to vertical (β) [deg]	0
Inclination factor for ultimate bearing capacity calculation (F_{ci})	1
Inclination factor for ultimate bearing capacity calculation (F_{qi})	1
Inclination factor for ultimate bearing capacity calculation ($F_{\gamma i}$)	1

Table 5.3 shows dependent variable values, which are calculated using formulas from chapters 2 and 3 and parameter values from Table 5.2, at the optimal design of the individual tower and foundation and the combined tower and foundation system formulations.

Table 5.3 Dependent Variable Values at Optimal Design

	Tower and Foundation	Tower and Foundation System
Variable (Symbol) [Units]	Value	Value
Taper of the tower (τ) [m/m]	0.006875	0.006875
Volume of the tower (V_t) [m^3]	34.7	34.8
Mass of the tower (m_t) [kg]	272,340	273,104
Volume of the foundation (V_f) [m^3]	187.6	187.6
Mass of the foundation (m_f) [kg]	450,156	450,156
Natural frequency (f_n) [Hz]	0.66666421	0.6664655
Max. princ. stress at C.P. A at tower top ($\sigma_1^{cpA\_t}$) [MPa]	13.6	13.54
Int. princ. stress at C.P. A at tower top ($\sigma_2^{cpA\_t}$) [MPa]	0	0
Min. princ. stress at C.P. A at tower top ($\sigma_3^{cpA\_t}$) [MPa]	-1.194	-1.191
Max. princ. stress at C.P. B at tower top ($\sigma_1^{cpB\_t}$) [MPa]	10.4	10.3
Int. princ. stress at C.P. B at tower top ($\sigma_2^{cpB\_t}$) [MPa]	0	0
Min. princ. stress at C.P. B at tower top ($\sigma_3^{cpB\_t}$) [MPa]	-6.77	-6.75
Max. princ. stress at C.P. A at tower base ($\sigma_1^{cpA\_b}$) [MPa]	185.8	185.3
Int. princ. stress at C.P. A at tower base ($\sigma_2^{cpA\_b}$) [MPa]	0	0
Min. princ. stress at C.P. A at tower base ($\sigma_3^{cpA\_b}$) [MPa]	-0.028	-0.028
Max. princ. stress at C.P. B at tower base ($\sigma_1^{cpB\_b}$) [MPa]	12.79	12.76
Int. princ. stress at C.P. B at tower base ($\sigma_2^{cpB\_b}$) [MPa]	0	0
Min. princ. stress at C.P. B at tower base ($\sigma_3^{cpB\_b}$) [MPa]	-4.54	-4.52
Mean cross-sectional radius (r_m) [m]	1.957	1.957
Elastic critical buckling stress ($\sigma_{CriticalElastic}$) [MPa]	2,289	2,295
Axial critical stress reduction coefficient (α_0)	0.666	0.666
Bending critical stress reduction coefficient (α_B)	0.729	0.729
Allowable local buckling stress ($\sigma_{buckling}$) [MPa]	290	290
Tip deflection at tower top (v_{top}) [m]	0.913	0.913
Tip rotation at tower top (v'_{top}) [deg]	1.053	1.052
Foundation depth (D_f) [m]	4.65	4.65
Equiv. surch. for soil above the foundation (q) [kN/m^2]	82.7	82.7
Foundation dimension b (b_f) [m]	3.07	3.07
Loading eccentricity of the foundation (e) [m]	2.03	2.03
Foundation effective area minor axis (b_e) [m]	7.67	7.67
Foundation effective area major axis (l_e) [m]	11.00	11.00

Table 5.3 continued

Foundation effective area (A_{eff}) [m^2]	61.5	61.5
Effective length of the foundation (L') [m]	9.39	9.39
Effective width of the foundation (B') [m]	6.54	6.54
Effective area of equivalent rectangle (A') [m^2]	61.5	61.5
Ultimate bearing capacity shape factor (F_{cs})	1.206	1.206
Ultimate bearing capacity shape factor (F_{qs})	1.123	1.123
Ultimate bearing capacity shape factor ($F_{\gamma s}$)	0.721	0.721
Ultimate bearing capacity depth factor (F_{cd})	1.158	1.158
Ultimate bearing capacity depth factor (F_{qd})	1.095	1.095
Ultimate bearing capacity depth factor ($F_{\gamma d}$)	1	1
Meyerhof ultimate bearing capacity (q_u') [kN/m^2]	480	480
Total ultimate load foundation can sustain (Q_{ult}) [MN]	29.5	29.5
Total vertical load on the foundation (Q_{total}) [MN]	9.84	9.84
Bearing capacity factor of safety ($FS_{bearing}$)	3	3
Maximum pressure on soil (q_{max}) [kN/m^2]	145.7	145.7
Max. pressure on soil factor of safety ($FS_{SoilPressure}$)	3.30	3.30
Rotational stiffness of the foundation (K_R) [GN-m/rad]	729	729
Horizontal stiffness of the foundation (K_H) [MN/m]	52,797	52,797
Sum of resisting moments ($\sum M_R$) [MN-m]	57.7	57.7
Sum of overturning moments ($\sum M_O$) [MN-m]	26.2	26.2

Table 5.4 shows the final height dependent variable values, which are calculated using formulas from chapters 2 and 3 and parameter values from Table 5.2, at the tower top and base for the individual tower and foundation and the combined tower and foundation system formulations.

Table 5.4 Final Height Dependent Variable Values at Tower Top and Base

Variable (Symbol) [Units]	Tower and Foundation		Tower and Foundation System	
	Value at Top	Value at Base	Value at Top	Value at Base
Height above ground (z) [m]	80	0	80	0
Extreme wind speed (V_{e50}) [m/s]	69.8	0	69.8	0
Wind pressure (q_i) [kg/(m-s^2)]	2,985	0	2,985	0
Outer diameter of tower (d_o) [m]	3.4	4.5	3.4	4.5
Horizontal distributed wind load (w_h) [kN/m]	6.79	0	6.79	0
Inner diameter of tower (d_i) [m]	3.33	4.43	3.33	4.43
Cross-sectional area of tower (A) [m^2]	0.373	0.495	0.374	0.496
Vertical distributed load (w_v) [kN/m]	29.5	38.9	29.6	39.0
Distance from tower top (y) [m]	0	80	0	80
Axial force at cross section (P) [MN]	1.342	4.08	1.342	4.09
Shear force at cross section (V) [MN]	0.810	1.320	0.810	1.320
Torque at cross section (T) [MN-m]	2.50	2.50	2.50	2.50
Moment at cross section (M) [MN-m]	2.73	97.2	2.73	89.1
Moment of inertia at cross section (I) [m^4]	0.528	1.233	0.529	1.236
Curvature in the vertical plane (v'') [m^{-1}]	3E-05	0.0004	3E-05	0.0004
Rotation in the vertical plane (v') [radians]	0.018	0	0.018	0.00003
Horizontal displacement (v) [m]	0.913	0	0.913	0.00003
2nd order moments (M_{2nd}) [MN-m]	0	2.79	0	2.80
Stress along the z-axis at C.P. A (σ_{ZcpA}) [MPa]	12.39	185.7	12.35	185.3
Stress along the z-axis at C.P. B (σ_{ZcpB}) [MPa]	3.60	8.24	3.59	8.24
Polar moment of inertia at cross section (J) [m^4]	1.055	2.47	1.058	2.47
Shear stress due to torque at C.P. A (τ_{ZXcpA}) [MPa]	4.03	2.28	4.02	2.27
1st moment of inertia at cross section (Q) [m^3]	0.20	0.35	0.20	0.35
Width where shear stress is calculated (b) [m]	0.07	0.07	0.07	0.07
Torque and shear stress at C.P. B (τ_{ZYcpB}) [MPa]	8.37	7.62	8.35	7.60
Max. principal stress at C.P. A (σ_1^{cpA}) [MPa]	13.58	185.8	13.54	185.3
Int. principal stress at C.P. A (σ_2^{cpA}) [MPa]	0	0	0	0
Min. principal stress at C.P. A (σ_3^{cpA}) [MPa]	-1.194	-0.03	-1.191	-0.03
Max. principal stress at C.P. B (σ_1^{cpB}) [MPa]	10.37	12.79	10.34	12.76
Int. principal stress at C.P. B (σ_2^{cpB}) [MPa]	0	0	0	0

Table 5.4 continued

Min. principal stress at C.P. B (σ_3^{cpB}) [MPa]	-6.77	-4.54	-6.75	-4.52
Effective stiffness ($\bar{k}$) [N-m]	836,543	0	840,170	109.1
Effective mass ($\bar{m}$) [kg-m^2]	47,677	0	47,884	0.00028

5.3.2 Numerical Data Obtained During Solution Process

Table 5.5 shows the constraint values at the optimal solution for the individual tower and foundation and the combined tower and foundation system formulations.

Table 5.5 Constraint Values at Optimal Solution

Constraint	Tower and Foundation Value	Tower and Foundation System Value
Lower limit on outer diameter of tower base (g_1) [m]	0.1	0.1
Upper limit on outer diameter of tower base (g_2) [m]	4.5	4.5
Lower limit on outer diameter of tower top (g_3) [m]	0.1	0.1
Upper limit on outer diameter of tower top (g_4) [m]	3.4	3.4
Lower limit on tower wall thickness (g_5) [m]	0.001	0.001
Upper limit on tower wall thickness (g_6) [m]	0.035261397	0.035361231
Lower limit on diameter of foundation (g_7) [m]	10	10
Upper limit on diameter of foundation (g_8) [m]	11.73425921	11.73425921
Lower limit on foundation thickness at outer edge (g_9) [m]	0.5	0.5
Upper limit on foundation thickness at outer edge (g_{10}) [m]	0.5	0.5
Limit on natural frequency (g_{11})	1.000003684	1.000003018
Allowable local buckling stress at tower top C.P. A (g_{12})	0.046872199	0.046726075
Allowable local buckling stress at tower base C.P. A (g_{13})	0.641158428	0.639310365
Allowable local buckling stress at tower top C.P. B (g_{14})	0.035777188	0.035664858
Allowable local buckling stress at tower base C.P. B (g_{15})	0.044128285	0.044027228
Maximum distortion energy at tower top C.P. A (g_{16})	0.002444699	0.002431282
Maximum distortion energy at tower base C.P. A (g_{17})	0.417567177	0.415470104
Maximum distortion energy at tower top C.P. B (g_{18})	0.002702237	0.002687301
Maximum distortion energy at tower base C.P. B (g_{19})	0.002929623	0.002916299
Tip deflection (g_{20})	0.913146973	0.913317194
Tip rotation (g_{21})	0.210539591	0.210344101
Limit on bearing capacity factor of safety (g_{22})	1	1
Limit on soil pressure factor of safety (g_{23})	0.910085954	0.910085954
Limit on minimum rotational stiffness (g_{24})	0.068586604	0.068586604
Limit on minimum horizontal stiffness (g_{25})	0.018940663	0.018940663
Limit on factor of safety against overturning (g_{26})	0.909249029	0.909249029

Table 5.6 shows the slack in the constraints at the optimal solution for the individual tower and foundation and the combined tower and foundation system formulations.

Table 5.6 Slack in Constraints at Optimal Solution

Constraint	Tower and Foundation Slack	Tower and Foundation System Slack
Lower limit on outer diameter of tower base (g_1) [m]	4.4	4.4
Upper limit on outer diameter of tower base (g_2) [m]	0	0
Lower limit on outer diameter of tower top (g_3) [m]	3.3	3.3
Upper limit on outer diameter of tower top (g_4) [m]	0	0
Lower limit on tower wall thickness (g_5) [m]	0.034261397	0.034361231
Upper limit on tower wall thickness (g_6) [m]	0.004738603	0.004638769
Lower limit on diameter of foundation (g_7) [m]	1.734259214	1.734259214
Upper limit on diameter of foundation (g_8) [m]	18.26574079	18.26574079
Lower limit on foundation thickness at outer edge (g_9) [m]	0	0
Upper limit on foundation thickness at outer edge (g_{10}) [m]	1	1
Limit on natural frequency (g_{11})	3.68437E-06	3.01811E-06
Allowable local buckling stress at tower top C.P. A (g_{12})	0.953127801	0.953273925
Allowable local buckling stress at tower base C.P. A (g_{13})	0.358841572	0.360689635
Allowable local buckling stress at tower top C.P. B (g_{14})	0.964222812	0.964335142
Allowable local buckling stress at tower base C.P. B (g_{15})	0.955871715	0.955972772
Maximum distortion energy at tower top C.P. A (g_{16})	0.997555301	0.997568718
Maximum distortion energy at tower base C.P. A (g_{17})	0.582432823	0.584529896
Maximum distortion energy at tower top C.P. B (g_{18})	0.997297763	0.997312699
Maximum distortion energy at tower base C.P. B (g_{19})	0.997070377	0.997083701
Tip deflection (g_{20})	0.086853027	0.086682806
Tip rotation (g_{21})	0.789460409	0.789655899
Limit on bearing capacity factor of safety (g_{22})	0	0
Limit on soil pressure factor of safety (g_{23})	0.089914046	0.089914046
Limit on minimum rotational stiffness (g_{24})	0.931413396	0.931413396
Limit on minimum horizontal stiffness (g_{25})	0.981059337	0.981059337
Limit on factor of safety against overturning (g_{26})	0.090750971	0.090750971

Table 5.7 shows the constraint status at the optimal solution for the individual tower and foundation and the combined tower and foundation system formulations. Constraints with a “Binding” status are active at the optimal solution.

Table 5.7 Constraint Status at Optimal Solution

Constraint	Tower and Foundation Status	Tower and Foundation System Status
Lower limit on outer diameter of tower base (g_1) [m]	Not Binding	Not Binding
Upper limit on outer diameter of tower base (g_2) [m]	Binding	Binding
Lower limit on outer diameter of tower top (g_3) [m]	Not Binding	Not Binding
Upper limit on outer diameter of tower top (g_4) [m]	Binding	Binding
Lower limit on tower wall thickness (g_5) [m]	Not Binding	Not Binding
Upper limit on tower wall thickness (g_6) [m]	Not Binding	Not Binding
Lower limit on diameter of foundation (g_7) [m]	Not Binding	Not Binding
Upper limit on diameter of foundation (g_8) [m]	Not Binding	Not Binding
Lower limit on foundation thickness at outer edge (g_9) [m]	Binding	Binding
Upper limit on foundation thickness at outer edge (g_{10}) [m]	Not Binding	Not Binding
Limit on natural frequency (g_{11})	Binding	Binding
Allowable local buckling stress at tower top C.P. A (g_{12})	Not Binding	Not Binding
Allowable local buckling stress at tower base C.P. A (g_{13})	Not Binding	Not Binding
Allowable local buckling stress at tower top C.P. B (g_{14})	Not Binding	Not Binding
Allowable local buckling stress at tower base C.P. B (g_{15})	Not Binding	Not Binding
Maximum distortion energy at tower top C.P. A (g_{16})	Not Binding	Not Binding
Maximum distortion energy at tower base C.P. A (g_{17})	Not Binding	Not Binding
Maximum distortion energy at tower top C.P. B (g_{18})	Not Binding	Not Binding
Maximum distortion energy at tower base C.P. B (g_{19})	Not Binding	Not Binding
Tip deflection (g_{20})	Not Binding	Not Binding
Tip rotation (g_{21})	Not Binding	Not Binding
Limit on bearing capacity factor of safety (g_{22})	Binding	Binding
Limit on soil pressure factor of safety (g_{23})	Not Binding	Not Binding
Limit on minimum rotational stiffness (g_{24})	Not Binding	Not Binding
Limit on minimum horizontal stiffness (g_{25})	Not Binding	Not Binding
Limit on factor of safety against overturning (g_{26})	Not Binding	Not Binding

Table 5.8 shows the Lagrange Multiplier (LM) values at the optimal solution for the individual tower and foundation and the combined tower and foundation system formulations. In Excel Solver, LM values for minimization problems are negative.

Table 5.8 Lagrange Multiplier (LM) Values at Optimal Solution

Constraint	Tower and Foundation LM Value	Tower and Foundation System LM Value
Lower limit on outer diameter of tower base (g_1) [m]	0	0
Upper limit on outer diameter of tower base (g_2) [m]	-	-
Lower limit on outer diameter of tower top (g_3) [m]	0	0
Upper limit on outer diameter of tower top (g_4) [m]	-	-
Lower limit on tower wall thickness (g_5) [m]	0	0
Upper limit on tower wall thickness (g_6) [m]	-	-
Lower limit on diameter of foundation (g_7) [m]	0	0
Upper limit on diameter of foundation (g_8) [m]	-	-
Lower limit on foundation thickness at outer edge (g_9) [m]	-62309.3	-62198.7
Upper limit on foundation thickness at outer edge (g_{10}) [m]	-	-
Limit on natural frequency (g_{11})	-405223	-407585
Allowable local buckling stress at tower top C.P. A (g_{12})	0	0
Allowable local buckling stress at tower base C.P. A (g_{13})	0	0
Allowable local buckling stress at tower top C.P. B (g_{14})	0	0
Allowable local buckling stress at tower base C.P. B (g_{15})	0	0
Maximum distortion energy at tower top C.P. A (g_{16})	0	0
Maximum distortion energy at tower base C.P. A (g_{17})	0	0
Maximum distortion energy at tower top C.P. B (g_{18})	0	0
Maximum distortion energy at tower base C.P. B (g_{19})	0	0
Tip deflection (g_{20})	0	0
Tip rotation (g_{21})	0	0
Limit on bearing capacity factor of safety (g_{22})	-15991.3	-15632.0
Limit on soil pressure factor of safety (g_{23})	0	0
Limit on minimum rotational stiffness (g_{24})	0	0
Limit on minimum horizontal stiffness (g_{25})	0	0
Limit on factor of safety against overturning (g_{26})	0	0

CHAPTER VI

DISCUSSION AND CONCLUSION

6.1 Discussion

The objectives of this research were two-fold: 1) to understand how optimization concepts and Microsoft Excel's optimization capabilities can be used in the design of wind turbine towers and foundations 2) to study how considering the tower and foundation as an integral system, where tower support conditions are not perfectly rigid, affects the optimal design. Results from this work show that optimization concepts and Excel can be used to obtain reasonable conceptual level designs and cost estimates for wind turbine towers and foundations. Additionally, formulating the design as an optimization problem allows the designer to more fully understand how various design parameters affect the optimal design and to efficiently develop site specific designs. Considering the tower and foundation as an integral system reduced the tower's natural frequency. This made the constraint on the tower's natural frequency more difficult to satisfy and resulted in a bulkier tower design.

This research extends the work of previous efforts to optimize wind turbine support structures in two primary ways. First, manufacturer provided tower top and foundation loads, which incorporate the current internationally accepted wind turbine design requirements outlined in IEC 61400-1, are used to obtain more realistic input for the structural analysis. Second, the foundation has been incorporated into the optimal design problem and its stiffness has been accounted for in calculating the tower's natural frequency.

Limitations of this work were primarily due to the limitations of Microsoft Excel's optimization solver and could be remedied by using a different solver. Excel Solver's Generalized Reduced Gradient method can only handle continuous problems. However, detailed wind turbine tower and foundation design is an inherently discrete problem (e.g. only certain plate thicknesses are available for the tower wall and towers are built from individual sections that typically vary in thicknesses from section to section instead of continuously over each section). Therefore, certain simplifications had to be made in order to accommodate the limitations of Solver. These simplifications limited the results of this research to the conceptual design level rather than the detailed design level. However, it is important to note that the detailed design level could be achieved by using an optimization solver capable of handling discrete problems.

One unexpected finding of this research was that considering the tower and foundation as an integral system resulted in a more expensive design. This finding was unexpected because previous research suggested that the opposite would occur. However, upon closer inspection, it is evident that the findings of this study are valid and that considering the foundational stiffness in natural frequency calculations will result in a more expensive design. More importantly, the results of this study suggest that tower designs that do not incorporate foundational stiffness effects may not be adequate. A fixed tower support condition assumes infinite foundational stiffness. Therefore, considering the foundational stiffness will automatically result in some decrease in stiffness. As stiffness decreases deflection increases. Since natural frequency varies with the square of deflection over deflection squared, an increase in deflection will result in a decrease in natural frequency. Thus, a bulkier design is required to satisfy the constraint on the minimum natural frequency of the tower. While the assumption of a fixed tower support condition may be satisfactory for stiff soils (e.g. clays), this assumption may not be valid for softer soils (e.g. sands) and should be questioned by engineers.

Sensitivity data (i.e. Tables 5.5 through 5.8) obtained during the solution process can be used to gain important insights into our problem. For instance, Table 5.7 shows that constraints on the upper limit of the outer diameter at the tower base, the upper limit of the outer diameter at the tower top, the lower limit of the foundation thickness at the outer edge, the limit on the natural frequency, and the limit on the bearing capacity factor of safety are active at the optimum. These active constraints have zero slack (ref. Table 5.6). In Excel, Lagrange Multiplier (LM) values are negative for minimization problems. However, since we are only interested in the relative magnitudes of the LM values, this is not a concern. It is important to note that Excel does not provide LM values for active upper bounds on design variables because these constraints are handled separately in the solution process for efficiency reasons. The LM values on the other active constraints show the benefit of relaxing a constraint and the penalty in tightening a constraint [Arora 2004]. However, before comparing, LM values for normalized constraints must be multiplied by the scale parameter used to normalize the constraint in order to obtain the true LM value. These final LM values are shown in Table 5.8. From which, it is observed that relaxing or tightening the limit on the natural frequency will result in the greatest benefit or penalty, respectively, on the optimum solution.

In summary, this work outlines in detail the process of transcribing a conceptual wind turbine tower and foundation design into an optimization problem and provides a general methodology that can be used to develop more sophisticated models. Additionally, it highlights the importance of considering the tower and foundation as an integral system and provides one example of how such a system could work in an optimization model.

6.2 Conclusions

Specific conclusions from this work include:

1. Optimization concepts and Microsoft Excel's optimization capabilities can be used to obtain reasonable conceptual level designs and cost estimates for wind turbine towers and foundations.
2. Detailed designs and cost estimates for wind turbine towers and foundations could be achieved using a solver capable of handling discrete optimization problems.

3. Considering the tower and foundation as an integral system results in a more expensive design. However, not considering the tower and foundation as an integral system may result in inadequate designs.
4. For the assumed parameter values shown in chapter 5, the constraint on the tower's natural frequency was found to control the tower design and the bearing capacity constraint was found to control the foundation design.
5. Relaxing or tightening the limit on the natural frequency will result in the greatest benefit or penalty, respectively, on the optimum solution.

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